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Connections between quark masses and quark-gluon condensates in a modified perturbative QCD

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Abstract: In this note, it is pointed out that the mass matrix for the six quarks can be generated in first approximation by introducing fermion condensates, on the same line as was done before for gluons, within the modified perturbative expansion for QCD proposed in previous works. Thus, the results point in the direction of the conjectured link of the approximate 'democratic' symmetry of the quark mass matrix and 'gap' effects similar to the ones occurring in superconductivity. The condensates are introduced here non-dynamically and the question of the possibility of their spontaneous generation therefore remains open. However, possible ways out of the predicted lack of 'democratic' symmetry of the condensates resulting from the spontaneous breaking of the flavour invariance are suggested. They come from an analysis based on the Cornwall-Jackiw-Tomboulis (CJT) effective potential for composite operators.

Keywords: Nonperturbative Effects, Confinement, Quark Masses and SM Parameters, QCD.

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JHEP04(2003)044

Contents

JHEP04(2003)044

1. Introduction

The nature of the chiral and flavour symmetries in QCD and the Standard Model is one of the open fundamental questions in high energy physics [1]–[6]. The amount of experimental information in direct relation with chiral and flavour physics that remains to be properly understood is enormous. This situation implies that these problems deserve to be considered along many directions in the search for a better understanding.

In previous works we studied the consequences of modifying the Feynman rules of perturbative QCD in a way that incorporates the presence of a condensate of zero-momenta gluons in the initial state for constructing the Wick expansion [7]–[10]. In [8] the modified theory included only a new term in the gluon propagator. The form of this change resulted from the adiabatic connection of the interaction on a modified vacuum of the non-interacting theory. The new state was chosen in a form similar to the way the BCS wave-function of superconductivity is sometimes represented, i.e. as an exponential of a quadratic polynomial in the creation operators. For the BCS wave-function, two electron creation operators are employed. In our discussion a quadratic polynomial in the gluon and ghosts creation operators were considered. The colourless character, and the additional requirement that the new vacuum be a physical state of the free theory in the non-interacting limit, strictly defined the form of the polynomial. Also, the Lorentz invariance requirement led us to consider only creation operators near the zero momentum limit.

The above propagator, which was employed in a simple tree calculation, directly led to a value of the gluon condensate parameter (defined as the mean value of the gauge lagrangian) different from zero. Moreover, when fixing the only new parameter C, to reproduce a current estimate for $\langle g^2G^2\rangle$, the one-loop corrections for the quark masses surprisingly gave one third of the nucleon mass value [9].

The mentioned elements naturally suggested the following idea: if the BCS-like modification of the gluon state could lead to such interesting conclusion, as the prediction of the constituent masses for quarks, it could be reasonable to expect that a similar state for the quarks, introduced in the massless QCD, could describe the lagrangian quark masses. This is the basic question addressed in this note.

The answer, at least at this preliminary stage, is positive. After modifying the free-quark propagator by introducing the zero-momentum terms representing the analogue to Cooper pair condensate in this problem (that is, all the colourless and uncharged quark-antiquark condensates), the simplest approximation for the Dyson equation for quarks produces a diagonal lagrangian mass matrix by the simple choice of a structure that is also diagonal, for the quark condensates. A second possibility is also analysed: it corresponds to the case in which the 'democratic' symmetry proposed by Fritzsch [2, 3] is assumed for the condensate matrix. It turns out that this symmetry is then translated to the quark mass matrix and the spectrum produces two massive quarks, each having two massless counterparts at the level of approximation considered. That is, the six initially equivalent quarks are decomposed in two triplets resembling the u and d type sets (u, c, t) and (d, s, b).

Further, the inclusion of the zero-momentum piece of the gluon propagator that reflects the gluon condensate is also examined, in order to check for consistency with the results obtained in [9] for the constituent masses, where the lagrangian quark masses were taken as parameters. The results for the six quarks were qualitatively in correspondence with the former evaluation. Again, the light quark masses turned out to have nearly 1/3 of the proton mass and the massive ones practically coincided with their lagrangian values. Thus, the picture seems to be consistent, up to these first approximations.

In ending the paper, comments are given about the possibility of justifying the pattern of quark condensates reproducing the quark masses, as generated by a dynamical breaking of the chiral symmetry. Possible solutions are suggested of the problem related with the indications of a necessary equality of the dynamically generated fermion condensates [5, 4]. The basic argument given here consists in first underlining the relevance (for the determination of the chiral symmetry breaking) of the gap equations considered in the CJT effective action approach of composite operators, and after that, to stress the need to include more than one-loop corrections to the CJT effective action for the quark and gluon propagators. These terms are directly connected with the existence of attractive channels between different fermions and should be of relevance in determining the strength of the condensate. They are linked with the two-particle Bethe-Salpeter propagator, whose associated bound state equation is in turn in close correspondence with the existence and strength of the condensate of Cooper pairs in the BCS description of superconductivity. On the other hand, the argument implying the equality of all the condensates rests on what could be, at first sight, a reasonable assumption, that, for a large number of colours, the one-loop correction to the effective action should be the leading contribution [5].

Finally, let us comment on the question of the gauge invariance of the proposed expansion. It can be remarked that this invariance should not be broken by the modified rules. The main points supporting this statement are: a) The Ward identities can be directly checked to be satisfied at tree level, and b) since the modified propagators (for massless

QCD) are exact inverse operators of the tree-level inverse kernels, it follows that the formal Wick expansion can be inverted to write it as a Feynman path integral over the gluon and quark fields. This fact indicates that the standard Ward identities should be valid in each order of perturbation theory. The concrete calculation of the one-loop polarization operator and other quantities done in [9, 10] confirmed this conclusion. However, the gauge covariant definition of the expansion for any value of the gauge parameter remains an important technical aspect to be considered (i.e. for a fully consistent implementation of the renormalization procedure) and it will be addressed elsewhere.

The paper is organized as follows: in section 2, the corrections to the one-loop quark self-energy and the related dispersion equations are discussed. section 3 is devoted to checking the effect of the additional consideration of the gluon-condensate term in the propagator. Finally, in this same final section, possible ways are suggested through which non-equal condensates for all the fermions could emerge dynamically.

2. Lagrangian quark masses in terms of condensates

As mentioned in the introduction, we will consider, within first approximation, the effects of employing a modified quark propagator that reflects the presence of quark-condensates in the vacuum of the free massless QCD. The specific form of the propagator will be

$$G_q^{i_1 i_2; f_1 f_2}(p) = \left(-\frac{\gamma^\mu p_\mu \delta^{f_1 f_2}}{p^2 + i\epsilon} + i\delta(p) C^{f_1 f_2} \right) \delta^{i_1 i_2}\,, \quad i, j = 1, 2, 3\,,$$

where the structure has a diagonal form in the colour indices, and the real and symmetric matrix in the flavour indices $C^{f_1 f_2}$ reflects the consideration of all possible sorts of colourless quark-antiquark condensates. As shown in refs. [8, 10] this zero-momentum contribution for fermions can be obtained after considering as the initial state, before the connection of the interaction, a modified vacuum in the form of an exponential of quadratic products of quark and antiquark creation operators as

$$| 0\rangle = \lim_{p^* \to 0} \exp\left(\sum_{f_1 f_2} \tilde{C}_q^{f_1 f_2}(p^*) \bar{q}^{+}_{f_1}(p^*) q^{+}_{f_2}(p^*) \right)\,, \tag{2.1}$$

in which the limit of a small null momentum $p^* \to 0$ should be taken in the corresponding infinite volume limit $V \backsim 1/ \mid \overrightarrow{p^*} \mid^3 \to \infty$, where $\overrightarrow{p^*}$ is the spatial part of p^*. The structure of this state is similar to the one employed for the representation of the BCS wave function; and the attractive and very strong character of the colour forces, at least naively, suggest the relevance of this approach to the description of QCD. Even in the presence of the weak phonon attraction with respect to the strong Coulomb repulsion between electrons, the Cooper pairs condensate becomes essential for the description of the superconducting state. Then, it is hard to think that, in the case of the attractive and even confining attraction due to colour interaction, the vacuum without condensates could be stable.

JHEP04(2003)044

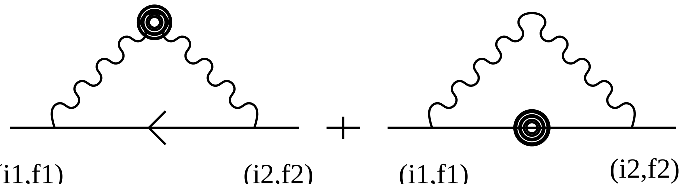

Figure 1: The condensate contributions to the one-loop self-energies. Note that the zero momentum Dirac delta functions anihilate the only existing integration. This circumstance suggests the existence of a systematic 'modified loop' approximation in the problem.

In order to consider the first implications of the introduced modification, lets us disregard for the moment the gluon condensate and only evaluate the contribution of the change in the quark propagator to the one-loop self-energy, associated to the second diagram in figure 1. The quark and gluon lines with central dots represent the added terms in the quark and gluon propagators. As the loop integration is annihilated by the delta function at zero momentum, the result is simply given by

$$\begin{aligned}\Sigma_q^{i_1 i_2;f_1 f_2}(p) &= \frac{\gamma_\mu \gamma^\mu g^2}{(2\pi)^4} T^a_{i_1 l} \frac{C^{f_1 f_2}}{p^2} T^a_{l\, i_2} \\ &= \frac{4g^2}{(2\pi)^4} T^a_{i_1 l} \frac{C^{f_1 f_2}}{p^2} T^a_{l i_2} \\ &= \frac{4g^2 C_F}{(2\pi)^4} \frac{C^{f_1 f_2}}{p^2} \,. \end{aligned} \tag{2.2}$$

where use has been made of the relations

$$\begin{aligned}(\gamma_\mu p^\mu)^2 &= p^2 \,, \\ T^a_{i_1 l} T^a_{l i_2} &= C_F \delta^{i_1 i_2} \\ C_F &= \frac{N^2-1}{2N} \,. \end{aligned}$$

Note that a better procedure could be to sum up all the mass insertions in the internal quark propagator joining the two vertices. However, here we are mainly interested in considering the simplest approximation. It should be noticed that this term can be viewed, in conjunction with its gluonic counterpart in figure 1 (to be introduced in the next section for treating gluon condensation), as the only two terms in a sort of 'modified tree' approximation for the self-energy in which no 'modified loop' radiative corrections are considered. The search for a systematic formulation for an expansion of this kind will be considered elsewhere. The relevance of such a loop expansion in determining an alternative and more rapidly convergent perturbative scheme is also suggested, for example, by the similar procedures in Bose condensation phenomena. There, the Bogoliubov shift in the amplitude of the scalar condensate field to the minimum, leads to a satisfactory perturbation expansion in terms of the new modified radiative field.

After substituting the self-energy approximate expression (2.2) in the Dyson equation for the propagator, and considering the quark wave modes solving the homogeneous version of the equation, it follows that

$$\begin{aligned} 0 &= \left(-p_\mu\gamma^\mu\delta^{i_1 i_2}\delta^{f_1 f_2} - \Sigma_q^{i_1 i_2; f_1 f_2}(p)\right)\Psi_{f_2}^{i_2}(p) \\ &= \frac{1}{p^2}\left(-p^2 p_\mu\gamma^\mu\,\delta^{f_1 f_2} - \frac{4g^2 C_F}{(2\pi)^4}\,C^{f_1 f_2}\right)\Psi_{f_2}^{i_1}(p)\,, \end{aligned} \tag{2.3}$$

which after being multiplied by the matrix between the parentheses, in which in addition the sign of the second term is changed, results in

$$\left(\left(p^2\right)^3\delta^{f_1 f_2} - \left(\frac{g^2 C_F}{4\pi^4}\right)^2\left(C^2\right)^{f_1 f_2}\right)\Psi_{f_2}(p) = 0. \tag{2.4}$$

It can therefore be noted that, within the considered approximation, the quark masses are defined by the eigenvalues of the square of the matrix of the condensates. Below, two phenomenological possibilities for fixing the condensate matrix will be treated.

2.1 Diagonal condensate matrix

This case corresponds to directly selecting $C_{f_1 f_2}$ as a diagonal matrix in the flavour indices, in such a way that its square, multiplied by $(\frac{g^2 C_F}{4\pi^4})^2$ and inserted in (2.4), will produce the mass matrix for the six quarks. Thus $C_{f_1 f_2}$ takes the form

$$g^2 C_{f_1 f_2} \equiv \frac{4\pi^4}{C_F}\begin{bmatrix} m_u^3 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_d^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_s^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_c^3 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_b^3 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_t^3 \end{bmatrix}, \tag{2.5}$$

which, after using the link between this matrix and the condensate values

$$\langle 0 \mid \Psi_{f_1}^i(0)\overline{\Psi_{f_2}^i}(0)^i \mid 0\rangle = \frac{4N}{(2\pi)^4}C^{f_1 f_2}\,, \tag{2.6}$$

leads to the following connection between the chiral condensates and the corresponding mass of the quark of flavour f:

$$g^2\langle 0 \mid \Psi_f^i(0)\overline{\Psi_f^i}(0)^i \mid 0\rangle = \frac{2(N^2-1)}{N}\,m_f^3,\ \ f = 1,\ldots,6. \tag{2.7}$$

The compatibility of these relations between quark condensates and masses with the existing extensive experimental and numerical information will be considered elsewhere. It seems particularly interesting to examine the connections of eq. (2.7) with the Gell-Mann-Oakes-Renner relation [11].

2.2 'Democratic' condensate matrix

The other selection to be analysed is the one that will produce as a result the 'democratic' symmetry (whose relevance was discussed in [2]) for each one of two separate quark triplets that we will call u- and d-type quarks. As has been argued [2], assuming that this first stage of the symmetry breaking can be justified dynamically, the masses for the other massless modes can then be produced at the next steps of approximation. It is thus interesting to verify that such a structure of the mass matrix can also be generated by the presence of quark condensates. The matrix $C_{f_1^* f_2^*}$, showing the symmetry in the above mentioned sectors, takes the form

$$g^2 C_{f_1^* f_2^*} \equiv \frac{4\pi^4}{3C_F} \begin{bmatrix} m_b^3 & m_b^3 & m_b^3 & 0 & 0 & 0 \\ m_b^3 & m_b^3 & m_b^3 & 0 & 0 & 0 \\ m_b^3 & m_b^3 & m_b^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_t^3 & m_t^3 & m_t^3 \\ 0 & 0 & 0 & m_t^3 & m_t^3 & m_t^3 \\ 0 & 0 & 0 & m_t^3 & m_t^3 & m_t^3 \end{bmatrix},$$

where the components of the flavour index vector used up to now, $f = u, d, s, c, b, t$, have been permuted to define the new flavour index $f^* = d, s, b, u, c, t$. This change was made in order to render the notation closer to the one employed in the literature for discussing these matrices. It can be noticed that the square, and even any kind of power, of this matrix also retains the same structure modulo constant factors. Thus, the mass matrix of the quarks, which can be obtained as being proportional to the power 1/3 of the matrix $C_{f_1^* f_2^*}$, also have the same form. Henceforth, the quark mass matrices proposed in [2] can also be generated by the presence of appropriate condensates in the vacuum having the same symmetry.

The matrix $C_{f_1^* f_2^*}$ above can be diagonalized by a unitary transformation U to produce, for the quark mass matrix:

$$M_{f_1^* f_2^*} = \left(\frac{g^2 C_F}{4\pi^4}\right)^{\frac{1}{3}} (U^{-1} C^{\frac{1}{3}} U)_{f_1^* f_2^*} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_t \end{bmatrix}.$$

Thus, the first steps in the approach related with the approximate 'democratic' symmetry of the experimental quark mass spectrum is also implied by the presence of the chiral condensates with the same symmetry as treated in the modified perturbative expansion.

3. Constituent masses from the gluon condensation

3.1 Generation of the constituent masses

Up to now, for the sake of clarity in this first exploration, we have considered only the new element with respect to the gluon condensate effects studied before. Let us now examine

JHEP04(2003)044

how the lagrangian quark masses, previously fixed in terms of the quark condensates, are modified by the introduction of the gluon condensate. The ideal situation would be that the results in this first step could reproduce the spectrum of masses obtained in [9] qualitatively. We will see that this is indeed what happens.

The modified propagator for the gluon has the form

$$G^{ab}_{\mu\nu}(p) = \left(\frac{1}{p^2 + i\epsilon} - i\delta(p)C\right)\delta^{ab}g^{\mu\nu}\,. \tag{3.1}$$

As can be observed, the Feynman gauge is used for the gluon propagator. This is because the modified rules were derived, in [8], precisely in this gauge in which the quantization of the free theory is simplified. However, as it was remarked at the end of the introduction, the extension of the rules to arbitrary values of the gauge parameter is under study. It should be underlined that this result is important for the technical completion of the modified expansion.

After inserting (3.1) in the expression associated to the second diagram in figure 1 for the quark self-energy, and integrating over the loop momentum, it follows that

$$\Sigma_g^{i_1 i_2; f_1 f_2}(p) = -\frac{M^2\delta^{i_1 i_2}\delta^{f_1 f_2}p_\mu\gamma^\mu}{p^2}\,, \tag{3.2}$$

where the value of the constant C determined in [9] from reproducing a current estimate for the gluon condensate parameter $\langle g^2G^2\rangle$ has been used. The constants that appear have the values

$$\begin{aligned} M &= \sqrt[2]{\frac{2C_F g^2 C}{(2\pi)^4}} = 333\,\mathrm{MeV/c^2}, \\ g^2C &= 65\left(\mathrm{GeV/c^2}\right)^2. \end{aligned} \tag{3.3}$$

Substituting the two contributions to the self-energy in the Dyson equation, the quark modes then satisfy

$$\frac{1}{p^2}\left(-p^2 p_\mu\gamma^\mu\left(1-\frac{M^2}{p^2}\right)\delta^{f_1 f_2} - \frac{4g^2C_F}{(2\pi)^4}C^{f_1 f_2}\right)\Psi_i^{f_2}(p) = 0\,, \tag{3.4}$$

which gives, again by taking the matrix obtained by changing the relative sign of the two terms in between the parentheses, and multiplying the equation by the result:

$$\frac{1}{p^2}\left((p^2)^3\left(1-\frac{M^2}{p^2}\right)^2\delta^{f_1 f_2} - \left(\frac{g^2C_F}{4\pi^4}\right)^2(C^2)^{f_1 f_2}\right)\Psi_i^{f2}(p) = 0\,. \tag{3.5}$$

It can be noticed that the quark dispersion relation is again determined by the square of the matrix of the quark condensates.

Let us now assume that the quark modes already diagonale the matrix $C^{f_1 f_2}$ with eigenvalues C_f for each flavour f. The new dispersion relations for the different quark polarizations associated to the eigenvalues C_f therefore become

$$p^2(p^2 - M^2)^2\delta^{f_1 f_2} - \left(\frac{g^2C_F}{4\pi^4}\right)^2 C_f^2 = 0\,. \tag{3.6}$$

JHEP04(2003)044

$Quark\ q$	$m_{qLow}^{Exp}(MeV)$	$m_{qUp}^{Exp}(MeV)$	$m_q^{Theo}(MeV)$
u	1.5	5	333–0
d	3	9	333–0
s	60	170	339–326–13
c	1100	1400	1255
b	4100	4400	4233
t	168600	17900	173500

Table 1: Solutions for the quark masses. The second and third columns indicate the lower and high experimental bounds of the quark lagrangian masses. The fourth one shows the results of solving the dispersion relations found here for the case of including both: gluon and quark condensates.

Further, consider the resulting mass spectrum under the following assumptions: a) the quark condensate parameters are fixed by determining the lagrangian mass matrix, as was done in the previous section, and b) the M constant has been fixed as before by reproducing the value of the gluon condensate. Then, the solution of the Dyson equation (3.6) determines the results for the pole quark masses in table 1, where the second and third columns give the low and high experimental bounds of the quark lagrangian masses. As can be observed from the data, the light quarks u, d and s have values close to 1/3 of the nucleon mass, as was also the outcome in [9]. Similarly also, the massive quark masses have remained almost invariant under the addition of the gluon condensate. The next salient feature of the results in the table is the fact that the strange quark (making honour to his name!) had three solutions at this approximation. One of them has the very low mass of 13 MeV, and the other two are in the same region as the value of the common mass (333 MeV) for the u and d quarks, but one lies over and the other below. It could be the case that, after introducing the above-mentioned sort of 'modified' one-loop approximation, the results can tend to approach even more those obtained in [9], where the lagrangian quark masses were introduced explicitly.

Finally, it must be remarked that various almost massless modes have been obtained. This outcome should be expected in the case when the symmetry has been spontaneously broken. Therefore, the appearance of massless modes is, at least, not in contradiction with the spontaneous generation of the quark condensates from the vacuum in a similar way, as the gluon condensate seems to be generated from the vacuum in first approximation [7], closely resembling the picture in the early Savvidi's chromomagnetic field approach.

3.2 Comment on the spontaneous generation of the quark condensates

Finally, the question of the spontaneous generation of the condensates from the vacuum will be analysed. Let us consider for concreteness the effective potential for composite operators introduced in [12] as applied to the massless QCD. This functional can be written in the following form

$$\begin{aligned} V[A,\Psi\Psi,G_g,G_q]_{(A,\Psi\Psi=0)} = {} & \frac{i}{2}\,\mathrm{Tr}[\log\left[G_g^{(0)-1}G_g - G_g^{(0)-1}G_g\right] - \\ & - i\,\mathrm{Tr}\left[\log[G_q^{(0)-1}G_q - G_q^{(0)-1}G_q\right] + \\ & + V^{(2)}\left[A,\Psi\Psi,G_g,G_q\right]_{(A,\Psi\Psi=0)}\ , \end{aligned} \tag{3.7}$$

JHEP04(2003)044

where V depends on arbitrary values of the mean quantum fields (here the quark and gluon ones) and G_g, G_q are also arbitrarily fixed expressions for their propagators. The $V^{(2)}$ functional represents contributions higher than one loop in terms of Feynman diagrams. The mean field values will be assumed to vanish in the ground state, as required by Lorentz invariance, and they will be omitted below. The free inverse propagators as considered in the previous section correspond to massless QCD and are given by

$$\begin{aligned}
G_q^{(0)-1} &\equiv -p_\mu \gamma^\mu, q = (u, d, s, c, b, t) \\
G_g^{(0)-1} &\equiv G_{g,\mu\nu}^{(0)-1ab}(p) = p^2 \delta^{ab} g^{\mu\nu} .
\end{aligned}$$

The traces in the fist term in (3.7) are in the spinor, fundamental colour and spatial indices; for the second term they are over the Lorentz, adjoint colour and spatial indices of the operators.

To analyse the spontaneous generation of the condensates from the vacuum, the propagator candidates showing non-vanishing condensates should be substituted in (3.7) in order to verify if their potential is or not lower than the one corresponding to the usual propagators. It could be helpful to recall that this should be the case, because the effective potential satisfies the condition of giving the minimum mean values of the energy in the subspace of the states of the system satisfying the constraints of having fixed the mean fields and propagators (which also are mean values).

It is interesting to notice that in the really first approximation for the propagators, in which only the delta function modifications are included, the first two terms in the above expression for the effective potential are completely independentof the values of condensate parameters. This is simply because the modifications introduced are also allowed free propagators for the theory, only differing from the Feynman ones in the rounding of the singularity. However, even at the simplest one-loop approximation, the propagator modes get masses and the mentioned two terms of the effective potential will depend on the selected condensation parameters.

The gluon and quark propagators, in the approximation in which the self-energies are defined by (3.2) and (2.2), take the form

$$\begin{aligned}
G_{g\mu\nu}^{ab}(p) &= \frac{1}{(p^2 + m_g^2)} \delta^{ab} g^{\mu\nu} , \\
G_q(p) &= \frac{\delta^{f_1 f_2}}{\left(-p_\mu \gamma^\mu (1 - \frac{M^2}{p^2}) - \frac{S_{f_1}}{p^2}\right)} ,
\end{aligned}$$

where the new parameters, which simplify the notation but also characterize the gluon and quark condensates, are related with the already defined constants C and $C_f, (f = u, d, s, c, b, t)$ through

$$\begin{aligned}
m_g^2 &= \frac{6 g^2 C}{(2\pi)^4} , \\
S_f &= \frac{g^2 C_F}{4\pi^4} C_f .
\end{aligned}$$

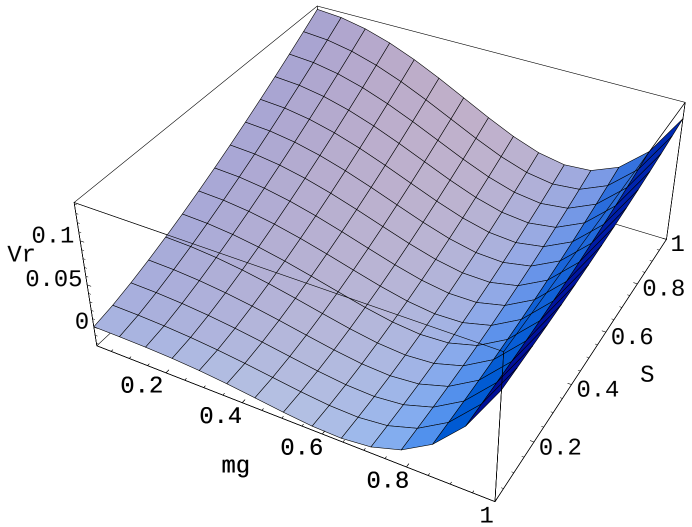

Figure 2: Effective potential as a function of the gluon and quark condensate parameters m_g^2 and S. The common value S is assumed for all six quark condensates. Note that there is no spontaneous quark condensation in this approximation.

The evaluation of various traces in the first two terms of (3.7) gives the following integral expression for this approximation of the CJT effective potential:

$$
\begin{aligned}
V[m_g, S_f] = -i \int \frac{dp}{(2\pi)^4} \sum_f \mathrm{tr} \Bigg[\log \left[p_\mu \gamma^\mu \frac{1}{p_\mu \gamma^\mu (1 - \frac{C_F}{3} \frac{m_g^2}{p^2}) + \frac{S_f}{p^2}} \right] - \\
-p_\mu \gamma^\mu \frac{1}{p_\mu \gamma^\mu (1 - \frac{C_F}{3} \frac{m_g^2}{p^2}) + \frac{S_f}{p^2}} \Bigg] + \\
+\frac{i}{2} \int \frac{dp}{(2\pi)^4} 3N \left[\log \left[\frac{p^2}{p^2 + m_g^2} \right] - \frac{p^2}{p^2 + m_g^2} \right] .
\end{aligned}
\tag{3.8}
$$

After deforming the integration paths of this expression so as to write it as an integral in euclidean space and renormalizing in the MS scheme (which consists in this case of a subtraction of a pole term being proportional to $m_g^4\, g^2 G^2$) the real part of the renormalized effective potential can be written in the form

$$
\begin{aligned}
V_r(m_g, S_f) &= 24\pi^2 \sum_f \int_0^\infty \int_0^1 \frac{d\lambda\, dq \lambda\, q^3}{(2\pi)^4} \times \\
&\quad \times \frac{f_1(q,\lambda,m_g,S_f) - f_2(q,\lambda,m_g,S_f)}{f_0(q,\lambda,m_g,S_f)} + \frac{153}{64\,\pi^2} m_g^4 \,\mathrm{Log} \frac{m_g^2}{\mu^2} \,, \\
f_1(q,\lambda,m_g,S_f) &= \left(s_f^2 - q^2 m_g^4\right) \left(\left(q^2 + m_g^2\right) \left(q^2 + m_g^4 (1-\lambda)\right) - s_f^2 (1-\lambda) \right) \\
f_2(q,\lambda,m_g,S_f) &= 2\, s_f^2 q^2 m_g^2 \left(\left(q^2 + 2m_g^2\right)(1-\lambda) + q^2 \right) \\
f_0(q,\lambda,m_g,S_f) &= \left(q^2 \left(q^2 + m_g^2\right)^2 + s_f^2 \right) \left(q^2 \left(q^2 + m_g^2 (1-\lambda)\right)^2 + (1-\lambda)^2 s_f^2 \right) .
\end{aligned}
$$

The dependence of V_r on the gluon and quark condensate parameters (assuming a common value S for the six quarks) is shown in figure 2. The evaluation of the imaginary

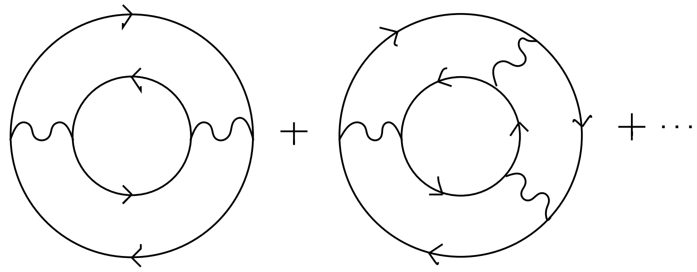

Figure 3: Types of two fermion-loop graphs contributing to $V^{(2)}$ which could be relevant to the determination of the pattern of spontaneous flavour symmetry breaking in massless QCD.

part, whose existence can reflect an instability of the condensate states, will be considered in an extension of this work.

It can be noticed that there is a spontaneous generation of the gluon condensate in this first one-loop approximation. The renormalization scale μ was fixed to the value in which the minimum of the potential for the pure gluon contribution occurs for a value of m_g^2 near to 0.5, which corresponds to $g^2 C = 64\,\mathrm{GeV}^2$. This is the value obtained by reproducing the estimated value of the gluon condensate parameter in previous works. However, the quark contribution displaces the minimum to slightly higher values. The plot also signals that, contrary to the case of gluons, the quark pair condensates are not spontaneously generated. By increasing the value of μ, it can also be seen that the minimum of the potential gradually tends to become independent of the magnitude of the fermion condensate S. Thus, it follows that the higher-loop pairing interactions could transform this flat behaviour on S in a one showing a minimum for non-zero values of the quark pair condensates, as will be mentioned below.

The fermion contribution evaluated here is the sum of identical functions of the quark parameters S_f. Therefore, if the expression could have developed a minimum outside the zero value of these parameters, it could have been in a point at which all these constants take identical values. This is the structure that had been predicted in ref. [5], from an analysis based on the limit of a large number of colours. In that work the central consideration was given to the plausibility that only the first-loop fermion correction should play a relevant role in the standard effective action of the system. However, in the presence of ground state or vacuum instabilities, the gap equations (which determine the real spectrum and ground state of the system and the possible field condensates), higher than one-, and in particular the two-loop fermion contributions, should play an important role. This can be the case because they are closely linked with the Bethe-Salpeter bound-state equations. Therefore, diagrams contributing to the $V^{(2)}$ term of the effective action, such as the ones illustrated in figure 3, can be expected to be relevant to the problem under consideration.

In another way, it is natural that this can be the situation in massless QCD, because the attractive binding forces are strong (unlike the case of the BCS superconductivity, where the attractive phonon force works against the Coulomb repulsion between electrons). In QCD, as the quark-antiquark pairs have attractive and strong colour forces, it would be expected that their condensates be spontaneously generated. But, as can be observed from the

JHEP04(2003)044

illustrated diagrams, the participation of different quark loops can introduce contributions that do not reduce to the sum of identical functions of each one of the parameters S_f. This fact seems to open the possibility for a dynamical generation of condensates that have no equivalent values for all the quarks. As within the modified expansion discussed here, the equality of all the condensates (within the considered simple approximation) is equivalent to the equality of the quark masses, the suggested possibilities for the spontaneous generation of different quark condensates would allow also the spontaneous generation of 'democratic' quark mass matrices, as proposed in [2]. If these remarks are appropriate, the question about the possible magnitudes of the condensates remains open as well. These issues should be considered in future extensions of this work. Finally, it can be useful to consider if there are any possible connections between the way the condensates are discussed here and other related pictures. One of them is the same older Nambu-Jona-Lasinio model [13]. Another is the 'top' condensate model for the Higgs field [14, 15, 16]. The resolution in the positive sense of the above stated dynamical problem for the generation of the quark pair condensates could open a way for giving a foundation to those pictures or other improved variations of them.

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JHEP04(2003)044